Introduction to Forces

1 Initial thoughts and motivation

1.1 Where are we now?

In our previous section we laid the groundwork for our study of physics, saying that we wished - as the first physicists in the Universe - to understand the world around us.

We embarked on this endeavour by starting to point out things that we observe, and in the process of doing so we realized that space does not look the same to everyone, and observations of the Universe will depend on who is observing. In terms of physics, this insight of relativity - *Galilean relativity* to be precise - is quite profound, that the Universe does not look the same to every observer. In the context of olympiads - which we are particularly interested in - we saw that this insight led to a particularly clever technique: whenever we are faced with a problem we can choose to view the system from the observer (frame of reference) according to which the system looks the simplest. This often makes a problem vastly simpler, and sometimes trivializes it completely. Finding which reference frame is the "simplest" is the tricky part of this technique, and there is no simple recipe for finding the best frame. Nevertheless - as is typical in problem solving - we learned that there are some guidelines ("heuristics") one can follow in trying to find these simple frames.

The better you get at finding these frames, the better problem solver you will become. For the purpose of all problems moving forward, I encourage you to always consider which frame you think would be the best to view this system in. Thereafter, please proceed with your calculations. For instance, if a system consists of only two objects but they are both moving, you may want to start by considering the system in the frame of one of the objects, such that there is only one moving part.

1.2 Newtonian mechanics

Now, having dealt with the question of accurately describing *what* is happening in the Universe, we get into the question of *why* or *how* things are happening. The very first attempt at formalizing observations into a complete system which can describe events in the Universe was made by Newton. I will assume here that you are familiar with Newton's laws from before, and will just simply state them here:

I. A body remains at rest or moving with constant velocity, if and only if there is no net force acting on it.

II. The net force on a body equals the body's acceleration times its mass ($F = ma$)

III. If one body exerts a force on a second body, then that second body always exerts a force equal in magnitude but opposite in direction on the first body.

You will likely have seen these before in your high school physics course, and so I will not delve into the details of them here, and instead I hope much of their nuances will gradually become clearer as we do more problems with these. Nevertheless, it is worth taking a step back and just admiring the simplicity of these laws, given the complexity of the physical phenomena that we still observe within mechanics.

Also, Newton's laws are centred around the idea of the ***force***. This is the fundamental core of his mechanical system. **Forces are vectors**, and as such we can sometimes solved problems just using the vectors in free space, which is a very nice property. Other than that however, the physical intuition behind forces is quite opaque (at least to me), and I think we are yet to find a really good intuition for what they are. You may consider it left as an exercise for the reader.

But, with Newton's laws in hand we are ready to start solving a whole host of mechanics problems. First, I will just briefly outline some typical forces that we come across in these problems.

1.3 Examples of forces

1.3.1 Gravity

In Newtonian mechanics, gravitational forces follow Newton "Universal" Law of Gravity. For simpler scenarios, such as those on Earth, we may consider the gravitational field to always be constant, hence:

$$F = mg$$

Bold here symbolizes that something is a vector. On Earth, g is $9.8ms^{-1}$ in the negative vertical direction (note that this is a vector).

1.3.2 Normal force

Normal force is just the force between two objects which are in contact with each other. Importantly, this forces is *always* perpendicular to the surfaces of the two objects at the point of contact. ¨

1.3.3 Friction force

The friction force arises due to the "rubbing" of rough surface against each other. Importantly, the firction force is proportional to the normal force between the objects. Hence we can write for friction for F_d

$$F_d = \mu N$$

where μ is the *coefficient of friction.*

1.3.4 Tension force

This is the force which works internally in something like a rope or a string, and which keeps it together. Tension is a tricky one to figure out (at least has been for me). I advise proceeding with caution and carefully applying Newton's laws rigorously when trying to find tension forces. I attached some questions below to get you to think about some non-intuitive facts about tension forces.

1.4 What can we do now?

I intend this handout as an introduction to problems with forces and also a bit of a refresher of high school (and high school-ish) physics problems. And so, what follows below is a bit of an esoteric array of problems which should introduce you to and hopefully get you comfortable with working with forces. The problems below only make use of the relatively standard forces outlined above in the example, and so no further knowledge is required.

Especially the *Check your understanding* are a little bit trickier than you may expect, and I would recommend them to anyone who wants to test their understanding of forces. The rest of the questions don't make use of any particular techniquee or trick, but should ideally be solvable without resorting to any fancy tricks like picking a clever reference frame, and rest more on just properly applying Newton's laws and solving equations of motion (possibly differentiating to find a minimum as in one of the questions). That said, the olympiad-style questions are certainly more tricky, but also do not rely on any tricks.

2 Questions

2.1 Check your understanding

1. Two people pull on opposite ends of a rope, each with a force F. What is the tension in the rope?

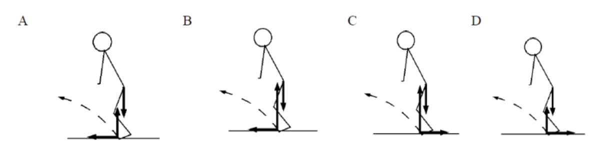

Figure 1: Forces in jump

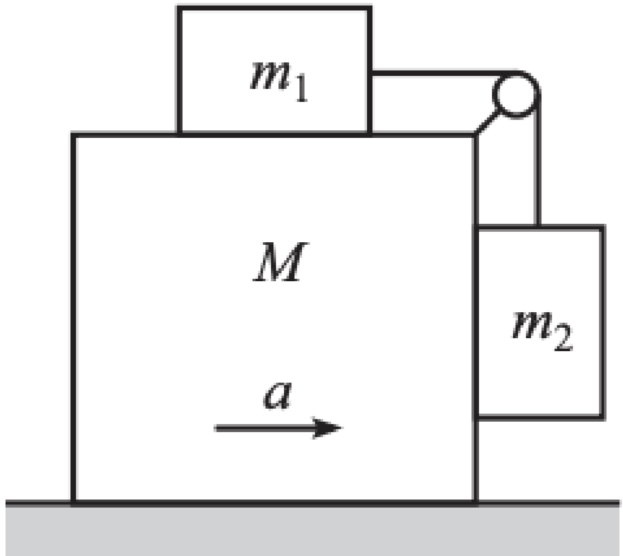

Figure 2: Masses on mass

2. Two teams, Brann and Viking, are competing in a tug-of-war. Brann pulls at the rope with a force of $5000N$, but Viking is currently winning (the rope is moving towards their end). Which of the following statements is correct?

   A. Viking is pulling at the rope with a force greater than 5000 N.

   B. Viking is pulling at the rope with a force of 5000 N.

3. A mass m rests on an incline with angle α (like in figure 7. What must the coefficient of friction be to stop the block from sliding down?

4. You're hopping upwards and forwards. Which of the figures in figure 1 most accurately shows the forces working on you?

2.2 Trying out the waters

1. All of the surfaces in the setup in figure 2 are frictionless. You push on the large block and give it an acceleration α. For what value of α is there no relative motion among the masses?

2. In Figure 3, if the box is stationary and the angle u between the horizontal and force is increased somewhat, do the following quantities increase, decrease, or remain the same: (a) F_x (horizontal force); (b) f_s (friction force); (c) F_N (normal force) (d) $f_{s,max}$ (maximum static friction force)?

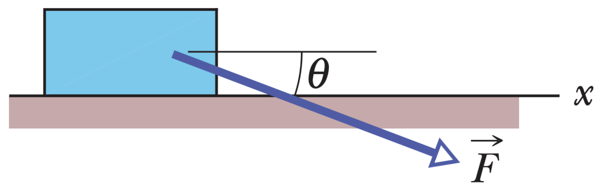

Figure 3: Box with force

3. A small body A starts sliding down from the top of a wedge whose base is equal to $l = 2.10m$. The coefficient of friction between the body and the wedge is $k = 0.140$. At what value of the angle α will the time of sliding be the least. What will it be equal to?

4. A block of mass M_1 sits on a block of mass M_2 on a frictionless table. The coefficient of friction between the blocks is μ. Find the maximum horizontal force that can be applied to (a) block 1 or (b) block 2 so that the blocks will not slip on each other.

5. A small body was launched up an inclined planeset at an angle $\alpha = 15°$ against the horizontal. Find the coefficient of friction, if the time of the ascent of the body is $\eta = 2.0$ times less than the time of its decent.

6. A book of mass M is positioned against a vertical wall. The coefficient of friction between the book and the wall is μ. You wish to keep the book from falling by pushing on it with a force F applied at an angle θ with respect to the horizontal $(-\pi/2 < \theta < \pi/2)$, as shown in Figure 4.

   a) For a given θ, what is the minimum F required?
   b) For what θ is this minimum F the smallest? What is the corresponding minimum F?
   c) What is the limiting value of θ, below which there does not exist an F that keeps the book up?

7. A block sits on a plane that is inclined at an angle θ. Assume that the friction force is large enough to keep the block at rest. What are the horizontal components of the friction and normal forces acting on the block? For what θ are these horizontal components maximum?

8. The inclined plane of figure 6 forms an angle $\alpha = 30°$ with the horizontal. The mass ratio $m_1/m_2 = \eta = 2/3$. The coefficient of friction between the body m_1 and the inclined plane $k = 0.10$. The masses of the pulley and the threads are negligible. Find the magnitude and the direction of acceleration of the body m_2 when the formerly stationary system starts moving.

2.3 Olympiad style questions

1. A small object is at rest on the edge of a horizontal table. It is pushed in such a way that it falls off the other side of the table, which is 1m wide, after 2 s. Does the object have wheels?

2. A ball is dropped from rest at height $4h$. After it has fallen a distance d, a second ball is dropped from rest at height h. What should d be (in terms of h) so that the balls hit the ground at the same time?

3. N identical uniform bricks of length L are stacked, one above the other, near the edge of a table. What is the maximum possible length the top brick can protrude over the edge of the table? How does this limit grow as N goes to infinity?

4. A bar of mass m is pull up by means of a thread up an inclined plane forming an angle α with the horizontal (see figure 5). The coefficient of friction is equal to k. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be minimum. What is it equal to?

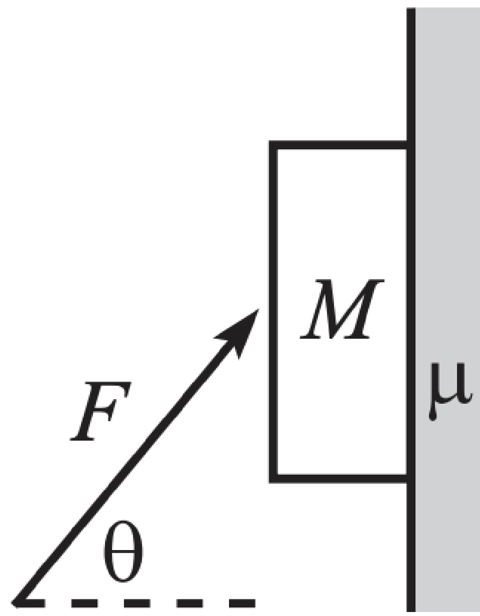

Figure 4: Book on wall

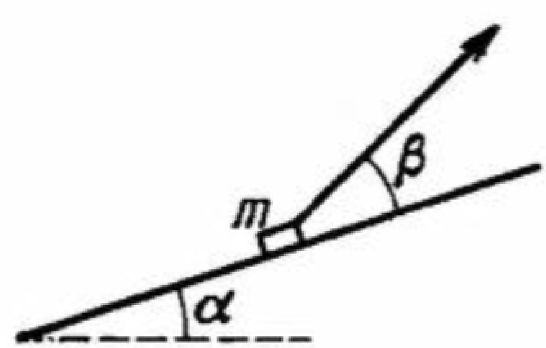

Figure 5: Mass pulled up on incline

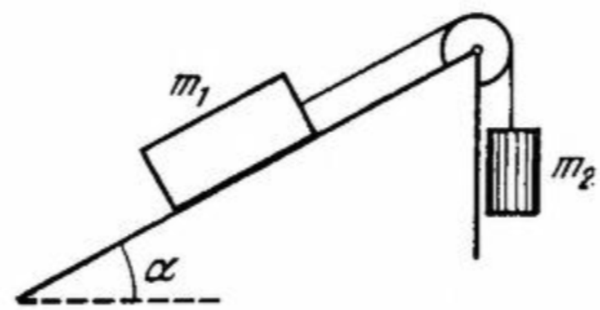

Figure 6: Masses with pulley on incline

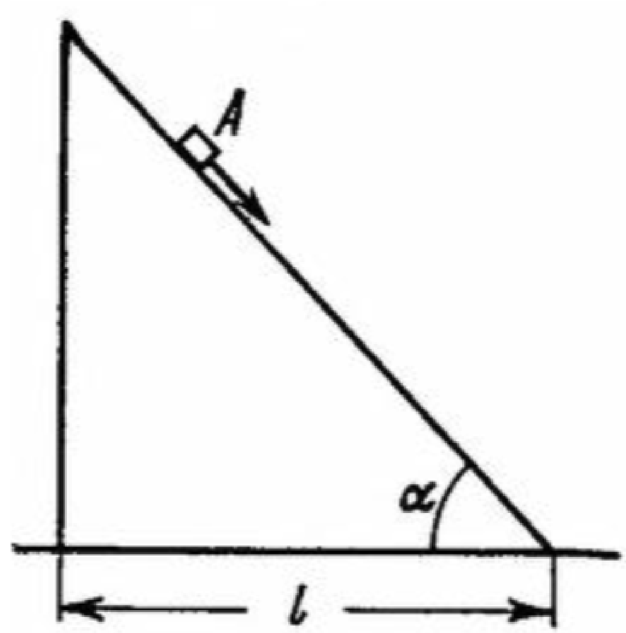

Figure 7: Mass sliding down incline